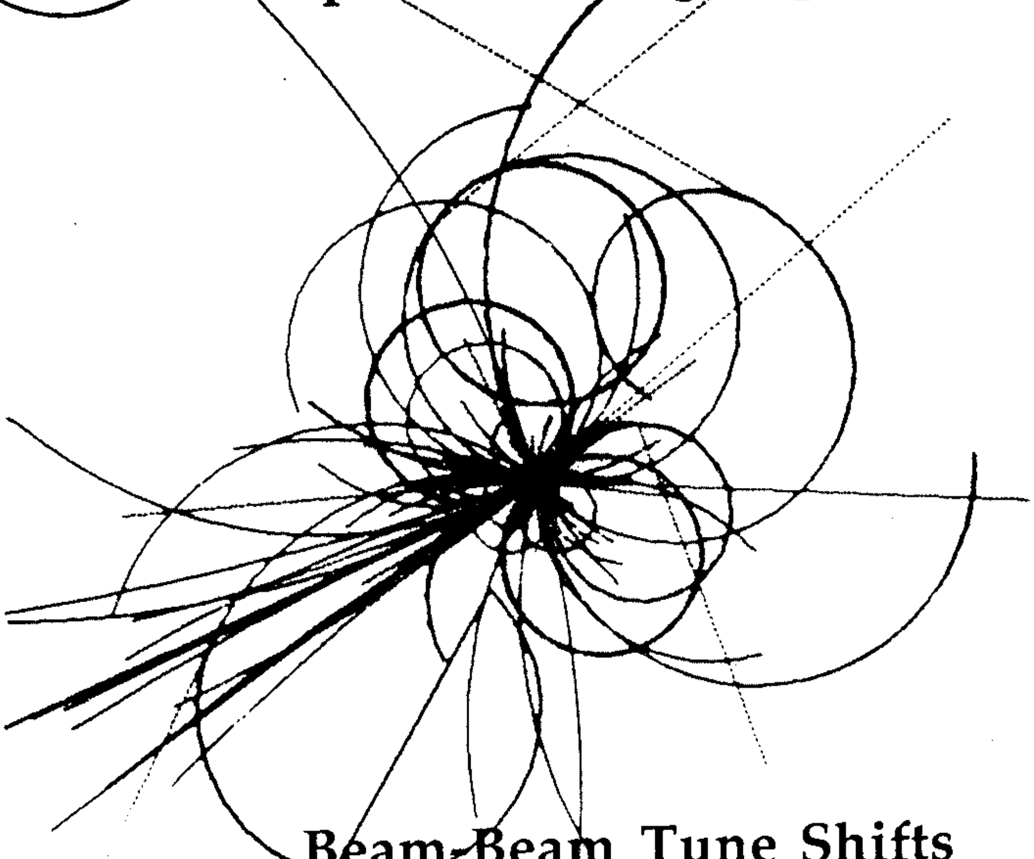
SSC-63

The Superconducting Super Collider



Beam-Beam Tune Shifts and Spreads in the SSC – Head On, Long Range, and PACMAN Conditions

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* Operated by Universities Research Association under contract with the U.S. Department of Energy

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INTRODUCTION

Operation of the SPS collider is limited by the nonlinear effects of the beam-beam interaction, with a peak performance which is expressed as a maximum total tune spread within the beam, estimated to be $\Delta\nu_{total} \leq 0.025$. Since the SSC is expected to have similar limitations it is desirable to keep the beam-beam dependent tune shifts and tune spreads small. In the SSC the beam-beam situation is complicated by the additional presence of long range interactions in the intersection regions (IRs) near the collision points, where the beams have not yet been separated into different beam tubes [1,2,3]. Furthermore, there may be systematic differences between the regions of the tune plane, (ν_x,ν_y), occupied by bunches near the edge of an abort gap in the collision train, and those in the middle of the train. The fear is that end bunches may have a reduced storage lifetime, so that they disappear, enlarging the abort gap and causing new bunches to be unstable in a self-regenerative process. This is called the PACMAN effect, after the (once) well known video game.

This note analyses the regions of the tune plane which are occupied in a (nominal) model SSC which has two collision points with $\beta^* = 0.5$ metres, and two with $\beta^* = 10.0$ metres [4]. The full crossing angle at each collision point is taken to be $\alpha^* = 75$ microradians, and the head on tune shift parameter is $\xi = -0.00084$, (significantly less than the SPS parameter $\xi_{SPS} \approx 0.004$). Long range beam-beam interactions take place in the free spaces on either side of the collision point, $\pm L^* = 72$ metres for the low beta IRs, and 150 metres in the high beta IRs, which are assumed for convenience to be pure drifts. Bunches in one beam are longitudinally separated from each other by $S_B = 4.8$ metres. In one variant of the model the transverse beam separation is vertical in all cases, while in a second variant alternate crossings are vertical and horizontal [5,6]. In both of these cases the horizontal and vertical tune shifts of a test particle in a nominal bunch, and in a PACMAN bunch, are calculated and plotted as a function of its transverse amplitudes, (a_x,a_y), which are conveniently measured in units of σ, the rms size of the incident gaussian beam. Similar results to these have also been reported elsewhere [7,8].

The exact expression for the tune shifts experienced by a test particle, due to head on collisions with round beams (which have the same gaussian density distribution in both transverse planes), is well known [9]. A multipole expansion which approximates the vertical and horizontal tune shifts due to long range collisions is presented below, and is shown to be accurate for amplitudes up to about half the beam separation when terms up to 16-pole are included. Both of these analytic expressions are incorporated in a Fortran program, SHIFT, which also includes a tracking option for cross checking the theoretical results. SHIFT takes as input a 'lattice' of six IRs, specified by ξ, L^*, S_B, and α^*, and as output produces total tune shifts for single pair of amplitude values, (a_x,a_y), or for a mesh of amplitude values.

HEAD ON INTERACTIONS AND TUNE SHIFTS

The beam-beam kick due to a single interaction with a round beam is written in complete generality as

$$\Delta x' = -(4\pi\xi/\beta)\,(2\sigma^2/r^2)\,(1-\exp(-r^2/2\sigma^2))\,.\,x \qquad 1.1$$

$$\Delta y' = -(4\pi\xi/\beta)\,(2\sigma^2/r^2)\,(1-\exp(-r^2/2\sigma^2))\,.\,(y+D_y) \qquad 1.2$$

with

$$r^2 = x^2 + (y+D_y)^2 \qquad 1.3$$

where x and y are the horizontal and vertical displacements of a test particle in one bunch from its own closed orbit, and D_y is the separation of the closed orbits of two bunches, arbitrarily assumed vertical. This is a non-linear perturbation of the linear particle motion around the lattice, given for example by

$$x = a_x\,\sigma(s)\,\sin(2\pi\nu_{x0}t + \phi(s)) \qquad 2$$

where t is the turn number and $\phi(s)$ is the betatron phase, which is a function of the azimuthal position, s, of the collision. Note that the test particle amplitude a_x, measured in units of the beam size, is a constant of the unperturbed motion. The resonance effects which restrict the operating point to small regions of the tune plane can cause large changes in this amplitude. However, since resonant behaviour is not the immediate topic of interest here, it is assumed that all resonances have been avoided, so that the amplitude remains, for all practical purposes, a constant of the perturbed motion.

In general the tune shift is given to first order in the perturbation strength, (for example horizontally), by

$$\Delta\nu_x = -(\beta/2\pi) . < \Delta x' \sin(\phi) > / a_x \sigma \qquad 3$$

where angle brackets $< >$ imply averaging over the betatron phase, ϕ. In the limit of small amplitudes, $a_x, a_y << 1$, the beam-beam kick in equation 1 becomes linear in x and z, exactly like a thin quadrupole. If the interaction is also assumed to be head on, $D_y = 0$, then

$$\{\Delta x', \Delta y'\} \approx -4\pi\xi/\beta \ . \ \{x, y\} \qquad 4$$

and by substitution of equations 4 and 2 into 3 it can be seen that both tunes are shifted by ξ, justifying its name as the 'tune shift parameter'.

If, instead, equations 1 and 2 are substituted into 3, the tune shift of a general amplitude test particle undergoing one head on collision per accelerator turn is given as

$$\Delta\nu_x/\xi = (1/2\pi^2) \int_0^{2\pi}\int_0^{2\pi} \frac{1 - e^{(-(a_x^2 s_x^2 + a_y^2 s_y^2)/2\sigma^2)}}{(a_x^2 s_x^2 + a_y^2 s_y^2)/2\sigma^2} \, s_x^2 \, d\phi_x \, d\phi_y \qquad 5$$

where $s_{x,y} \equiv \sin\phi_{x,y}$. This can also be written as

$$\Delta\nu_x/\xi = \int_0^1 e^{-(\alpha_x + \alpha_y)u} \, I_0(\alpha_x u) \, \left[I_0(\alpha_x u) - I_1(\alpha_x u)\right] du \qquad 6$$

where $\alpha_{x,y} \equiv (a_{x,y}/2)^2$, and where I_0 and I_1 are reduced Bessel functions. Equation 5 or 6 can also be expanded as a multipole expression for the head on tune shift,

$$\begin{aligned}\Delta\nu_x/\xi = [\ 1 &- (1/4) \ \{ (3/4) A_x^2 + (1/2) A_y^2 \} \qquad 7 \\ &+ (1/16) \ \{ (5/12) A_x^4 + (1/2) A_x^2 A_y^2 + (1/4) A_y^4 \} \\ &- (1/64) \ \{ (35/192) A_x^6 + (15/48) A_x^4 A_y^2 + (9/32) A_x^2 A_y^4 + (5/48) A_y^6 \} \\ &+ \ \ldots\ldots\ldots\ ]\end{aligned}$$

which contains terms upto 16-pole order, and is valid for small amplitudes. Similar expressions for the vertical tune shift are obtained merely by interchanging x and y in equations 5, 6, and 7.

Figures 1.1 and 1.2 plot the behaviour of the tune shifts predicted by equations 5 and 6, as functions of the amplitudes. The first figure, 1.1, shows the relative tune shifts as fractions of ξ, while figure 1.2 shows the absolute tune shifts due a single head on collision per turn of strength $\xi = -0.00084$, nominal for the SSC.

LONG RANGE TUNE SHIFTS AND SPREADS

At a distance L from an interaction point, in the absence of quadrupoles, the beta function and the beam size are given by

$$\beta = \beta^* \,(1 + (L/\beta^*)^2) \approx L^2/\beta^* \qquad 8.1$$
$$\sigma \equiv (\varepsilon\beta/\gamma)^{1/2} \approx \sigma^* \,(L/\beta^*) \qquad 8.2$$

where ε is the normalised emittance, γ is the usual relativistic factor, and where a superscript * refers to the value of a variable at the collision point. The approximations are valid at large distances, $L >> \beta^*$. When equation 8.2 is inserted into the definition of the tune shift parameter,

$$\xi \equiv -N r_p \beta \,/\, 4\pi\gamma\sigma^2 = -N r_p \,/\, 4\pi\varepsilon \qquad 9$$

(where N is the number of protons in the incident bunch, and r_p is the classical radius of the proton) it is seen that ξ is idependent of both β and the energy. Hence the tune shift parameter has exactly the same value for all (round) beam-beam interactions, regardless of which intersection region is involved, of whether the interactions are head on or long range, or of whether they occur at injection or at storage energies. Note that ξ is negative for repulsive bunches of the same charge.

When normalised coordinates are introduced (for example horizontally), through

$$x_n \equiv x/\sigma \qquad 10$$
$$x_n' \equiv (\alpha_t\, x + \beta\, x')\,/\,\sigma \qquad \text{so that} \qquad \Delta x_n' = \beta/\sigma \,.\, \Delta x'$$

where α_t is a Twiss parameter, the unperturbed (horizontal) motion describes a circle in x_n, x_n' phase space. The connection between head on and long range tune shift behavior is drawn even closer when equations 10 are substituted into 1, with the equivalent vertical equations. After dropping the subscript $_n$, the exact form of the (horizontal) normalised beam-beam interaction then becomes

$$\Delta x' = -8\pi\xi . x . [1-\exp(-(x^2+(y+d_y)^2)/2)] \;/\; [x^2+(y+d_y)^2] \qquad 11$$
$$\Delta y' = -8\pi\xi . (y+d_y) . [1-\exp(-(x^2+(y+d_y)^2)/2)] \;/\; [x^2+(y+d_y)^2]$$

The normalised vertical beam separation d_y used in these expressions is the total separation angle divided by the rms angular size of the beam,

$$d_y \equiv D_y/\sigma = \alpha^* L/\sigma \approx \alpha^* \beta^* / \sigma^* \qquad 12$$

so that for nominal SSC conditions, with $\varepsilon = 10^{-6}$ metres, $\gamma = 2.10^4$, and a total crossing angle of $\alpha^* = 75$ microradians,

$$d_y \approx \alpha^* (\beta^* \gamma/\varepsilon)^{1/2} = 7.5 \; (2\beta^*)^{1/2} \qquad 13$$

Since d_y becomes constant in the large distance limit, $L >> \beta^*$, and recalling that the normalised amplitudes, a_x, are constants of the motion, then equation 11 shows that all long range collisions behave in the same general way, and in particular cause the same tune shifts. Combining all $4L^*/S_B$ long range collisions in one intersection region into one single equivalent interaction, then, and assuming that the separation is large, $d >> 1$, so that the exponential term can be dropped,

$$\Delta x' = 4\pi\, \xi_{LR} . \; x \;/\; [(x/d_y)^2+(1+y/d_y)^2] \qquad 14.1$$
$$\Delta y' = 4\pi\, \xi_{LR} . (y+d_y) \;/\; [(x/d_y)^2+(1+y/d_y)^2] \qquad 14.2$$

where it is convenient to introduce the long range tune shift parameter,

$$\xi_{LR} \equiv 4L^*/S_B . 2/d_y^2 . \xi = 0.0014 . L^*/d_y^2 \qquad 15$$

The expression which is produced when equations 14.1 and 14.2 are expanded in the small amplitude limit, $a_x/d_y, a_y/d_y << 1$,

$$\{\Delta x', \Delta y'\} \approx 4\pi\, \xi_{LR} . \{x, \; d_y - y\} \qquad 16$$

is analogous to equation 4, implying that for vertically separated beams of oppositely charged particles there is a positive long range horizontal tune shift of magnitude ξ_{LR}, accompanied by a negative vertical tune shift of the same size. There is also a constant vertical (dipole) kick, which shifts the closed orbit of the test particle, but which for present purposes can be ignored.

If equations 14.1 and 14.2 are expanded as multipoles up to 16-pole order, and substituted into equation 3, the tune shifts as functions of the amplitudes are given as

$$\begin{aligned}\Delta\nu_x = \xi_{LR} \cdot \Big[& -1 + \{ (3/4)\,A_x^2 - (3/2)\,A_y^2 \} \\ & - \{ (5/8)\,A_x^4 - (15/4)\,A_x^2A_y^2 + (15/8)\,A_y^4 \} \\ & + \{ (35/64)\,A_x^6 - (105/16)\,A_x^4A_y^2 + (315/32)\,A_x^2A_y^4 - (35/16)\,A_y^6 \} \\ & - \;\ldots\ldots\ldots\ldots \Big]\end{aligned} \qquad 17.1$$

$$\begin{aligned}\Delta\nu_y = \xi_{LR} \cdot \Big[& +1 + \{ (3/4)\,A_y^2 - (3/2)\,A_x^2 \} \\ & + \{ (5/8)\,A_y^4 - (15/4)\,A_y^2A_x^2 + (15/8)\,A_x^4 \} \\ & + \{ (35/64)\,A_y^6 - (105/16)\,A_y^4A_x^2 + (315/32)\,A_y^2A_x^4 - (35/16)\,A_x^6 \} \\ & + \;\ldots\ldots\ldots\ldots \Big]\end{aligned} \qquad 17.2$$

where it is convenient and natural to use relative amplitudes scaled by the beam separation,

$$A_x \equiv a_x/d_y \qquad \text{and} \qquad A_y \equiv a_y/d_y \qquad 18$$

If the beam separation is horizontal instead of vertical, the correct results follow by interchanging x and y everywhere.

Figure 2.1 shows the relative tune shifts, $\{\Delta\nu_x/\xi_{LR}, \Delta\nu_y/\xi_{LR}\}$, for a range of the relative amplitudes, $\{A_x, A_y\}$, from $\{0, 0\}$ to $\{1.5, 1.5\}$, as calculated by using the tracking option in the program SHIFT. It shows that both tune shifts tend to zero as A_x and/or A_y tend to infinity, as expected, and that the largest tune shift of about $-7.5\ \xi_{LR}$ occurrs horizontally, when $\{A_x, A_y\} \approx \{0.0, 0.9\}$, that is, when the vertical oscillation amplitude is close to the vertical beam separation.

Figure 2.2 compares the absolute tune shifts for the nominal vertical separation case, with $(\beta^*, L^*) = (0.5, 72)$ metres, and $(d_y, \xi_{LR}) = (7.5, 1.792\ .10^{-3})$, as they have been obtained by tracking, and by using the multipole approximations given in equations 17.1 and 17.2 above. It shows that the analytic approximation truncated at 16-pole order agrees well with tracking for amplitudes $a \le 4$, that is, for amplitudes up to about half the separation. These amplitude limits are respected in all of the results which are presented below, justifying the use throughout of the analytical forms for head on and long range tune shifts, as given by equations 6 and 17 .

RESULTS

Figure 3.1 shows tune shift data for purely long range interactions under the nominal conditions used above, $(\beta^*, L^*) = (0.5, 72)$ metres, and $(d, \xi_{LR}) = (7.5, 1.792\ .10^{-3})$, with maximum amplitudes of four times the rms beam size. The beams were separated horizontally in one case, and vertically in a second case, forming two distributions which are symmetric about the line $x = y$, with a spread which is much less then than the tune shift from the origin.

Figure 3.2 shows the equivalent long range tune spreads and shifts near a high beta collision point, with $(\beta^*, L^*) = (10.0, 150)$ metres, and $(d, \xi_{LR}) = (33.5, 1.867\ .10^{-4})$, with a much larger normalised separation, d. Both ξ_{LR} and the octupole order tune shift divided by ξ_{LR} vary with d^{-2}, so that the larger separation not only causes the tune shift ξ_{LR} to decrease by almost an order of magnitude, but also causes the tune spread between $a_x, a_y = 0,0$ and $4,4$ to become almost invisible.

Figure 4.1 shows the net tune shifts and spreads, when the components due to all the head on and long range interactions of a nominal SSC, with two low beta (0.5 metre) and two high beta (10.0 metre) interaction regions, are added together. The beam separation is vertical in all interaction regions. It also shows what happens to a PACMAN bunch which has all the nominal head on collisions, but has only half of the nominal number of long range collisions, in all of the interaction regions. Not only is the tune shift much larger than the tune spread in this figure, dominated by the long range interactions, but the PACMAN behaviour also increases the effective operational tune spread. While it will be possible to adjust the two tunes of the SSC to compensate for the tune shift, this effective spread may not be compensated.

The net tune shift may be greatly reduced by arranging that pairs of successive interaction points, with identical collision betas, occur with alternating vertical and horizontal separation. In this case the long range tune shifts of a test particle with vanishingly small amplitudes will cancel, since the long range interaction is defocussing in the plane of separation, and focussing in the other plane with an identical strength. This leaves only the head on tune shifts to accumulate, even for the PACMAN bunches (as they have been defined here). Figure 4.2 shows the tune shifts which result from the same model considered in figure 4.1, but now with alternating separation in the high and low beta interaction regions, so that the four plots in figures 3.1 and 3.2 are added together to get the net (long range) effect. Not only is its tune shift greatly reduced, but the tune spread of the PACMAN bunch lies roughly on top of the spread of the nominal bunch. While the size of the nominal spread has barely changed, the effective spread of the beam has been significantly reduced.

CONCLUSIONS

In the nominal SSC conditions which have been studied, the tune shifts and spreads due to head on beam-beam interactions are generally comparable in size to those due to long range beam beam interactions. The two kinds of interactions behave differently as functions of the two transverse amplitudes, however, with the head on tune shifts and spreads increasing with decreasing amplitudes, while the long range tune spread decreases with increasing amplitudes. Long range tune shifts are somewhat insensitive to varying amplitudes, with the largest contributions coming from the low beta intersection regions. The total beam beam tune spreads are significantly smaller than those which have been found operationally acceptable in the SPS. However, the tune shifts in the standard case of vertical separation at all crossing points are much larger than the spreads, and are comparable to values in the SPS. While this is not a critical issue for operation of the SSC, since the tune shifts can be compensated by adjusting the unperturbed tunes, it is nevertheless worth examining ways to weaken the net effect of the long range interactions.

One way to alleviate the situation is to increase the crossing angle, α^*, since this decreases the scaled separation d like $\alpha^{*\,-1}$, reducing the long range tune shift and (octupole order) tune spread like $\alpha^{*\,-2}$. The comparative irrelevance of the tune shifts caused by long range beam beam interactions when d is increased from its nominal value of 7.5 to 33.5, in the case of 10.0 metre beta intersection regions, has already been illustrated above, in figure 3.2 . However, the crossing angle may only be increased subject to the following limitations. First, the crossing angle must not be much larger than the characteristic angle of the bunch, $\alpha_0 \equiv 2\sigma_{width}/\sigma_{length}$, which is nominally about 64 microradians when the length is 0.15 metres. It is well known that violation of this condition results in luminosity loss, odd order beam-beam resonances, and synchrobetatron satellites [10-13]. Second, the beam separation at the intersection region triplet, $\alpha^* L_{triplet}$, which is centered about 40 metres away from the collision point in the 0.5 metre case, may not be larger than a few millimetres. In view of these two constraints, the nominal solution with a 75 microradian crossing angle therefore appears to be well optimised already, with head on and long range non linearities roughly equal in strength.

A more promising solution is to make the crossings at pairs of identical intersection regions alternately horizontal and vertical. This reduces the long range tune shift to zero, but leaves the long range tune spread essentially unchanged. It has the further advantage of decreasing the additional effective tune spread due to PACMAN bunches, which have different tune shifts than the nominal bunches. Such a scheme should avoid complicating the layout of the lattice, and keep the two rings rigorously one above the other. This appears to be possible by the use of moderate strength splitting dipoles, in both planes, placed beyond the intersection region triplet but before the beams enter separate rings.

Finally, it should be noted that the presence of quadrupoles in the intersection region, contrary to the assumption which has been made that all long range collisions take place in a drift region, does not threaten the validity of the results which have been presented. It can be shown that the approximations which have been made are satisfactory if the betatron phase advance over the region is negligible. This is certainly true over the range from, say, $L = 2\beta^*$ to L^*, in both of the cases which have been studied. A more significant error is the mistreatment of the collisions which are not distant, where $L \approx \beta^*$. However, in the low beta (0.5 metre) case there are no such collisions, and the four or five such collisions in the high beta (10.0 metre) case do not change the qualitative observation that as a whole the high beta long range collisions are unimportant.

ACKNOWLEDGEMENTS

We are glad to acknowledge the many useful discussions on this subject which we have had with Alex Chao and Sam Heifets, whose comments have been incorporated in the body of the text.

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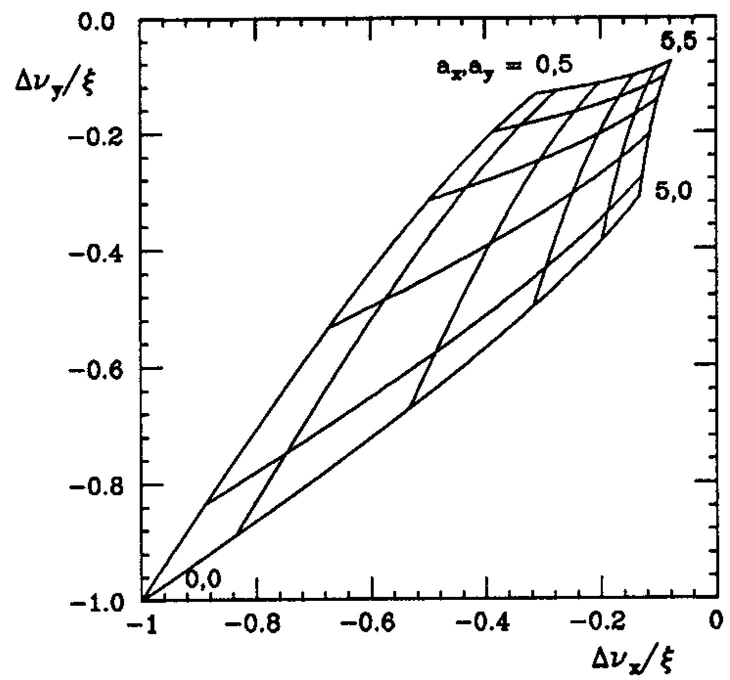

Figure 1.1 The relative tune shifts experienced by a test particle, in a head on collision with a round gaussian beam of the same charge sign. The horizontal and vertical amplitudes, a_x and a_y, are measured in units of the rms beam size, σ.

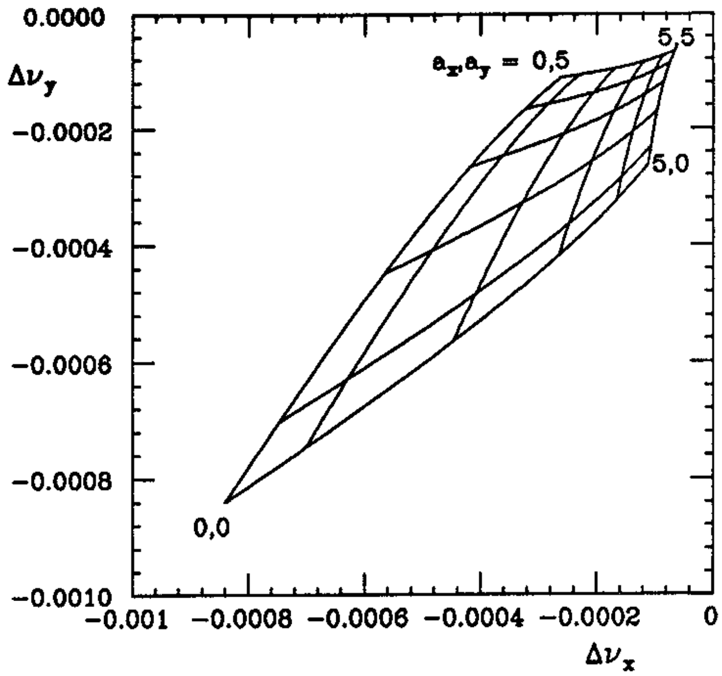

Figure 1.2 The absolute tune shifts due to a single nominal head on collision with $\xi = -8.4 \times 10^{-4}$, in a proton-proton SSC, over a mesh of amplitudes, a_x, a_y, which are measured in units of the beam size, σ.

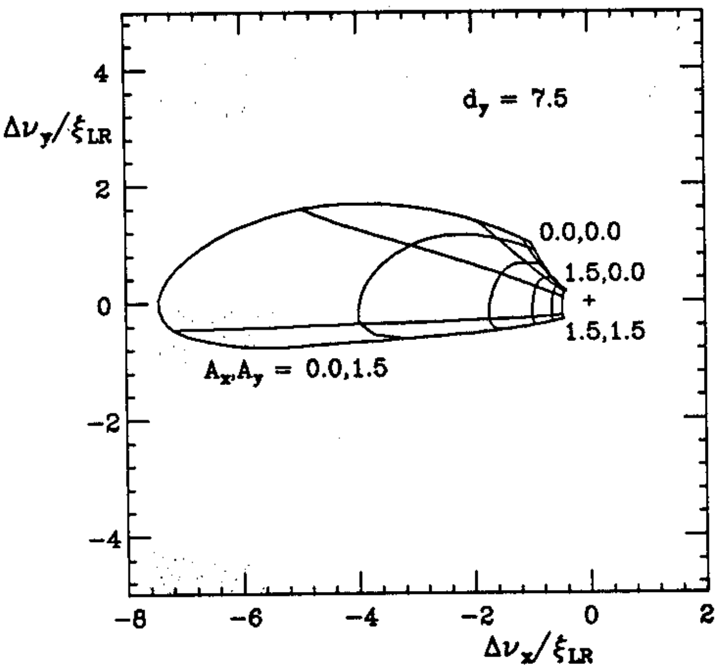

Figure 2.1 The relative tune shifts due to a long range collision, or a series of contiguous collisions in a drift, measured in units of the long range tune shift parameter, ξ_{LR}. Amplitudes are measured as fractions of the vertical beam separation, 7.5 σ.

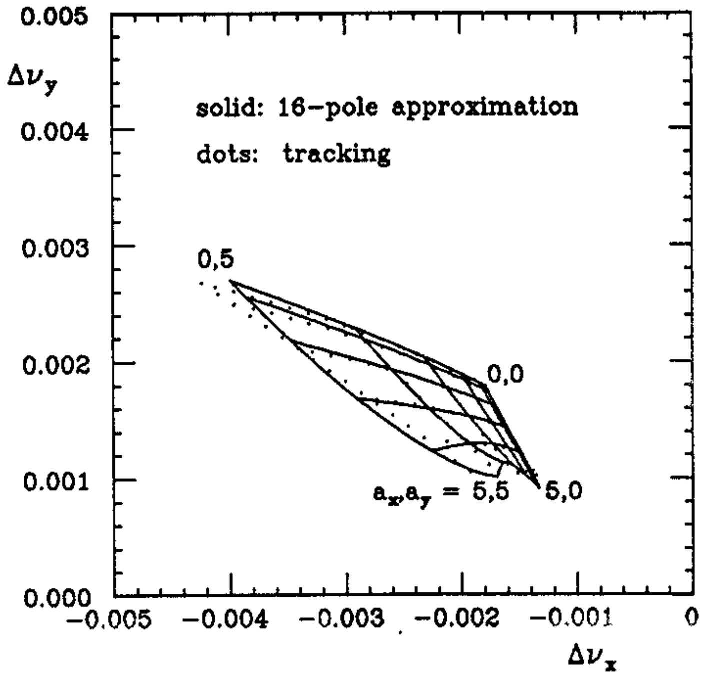

Figure 2.2 The absolute tune shifts due to long range collisions between beams vertically separated by 7.5 σ, over 72 metres on either side of an interaction point with $\beta^* = 0.5$ metres. The bunches are spaced longitudinally by 4.8 metres, with $\xi = -8.4 \times 10^{-4}$, so that $\xi_{LR} = 1.792 \times 10^{-3}$.

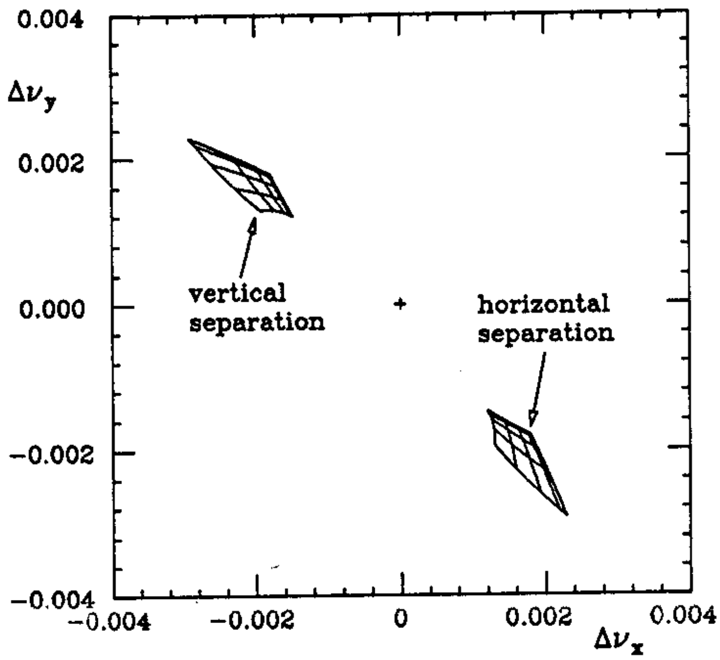

Figure 3.1 Long range tune shifts and spreads over a mesh of amplitudes up to 4σ in each plane, due to encounters in a nominal low beta (0.5 metre) intersection region with a vertical, or horizontal, beam separation of $7.5\ \sigma$.

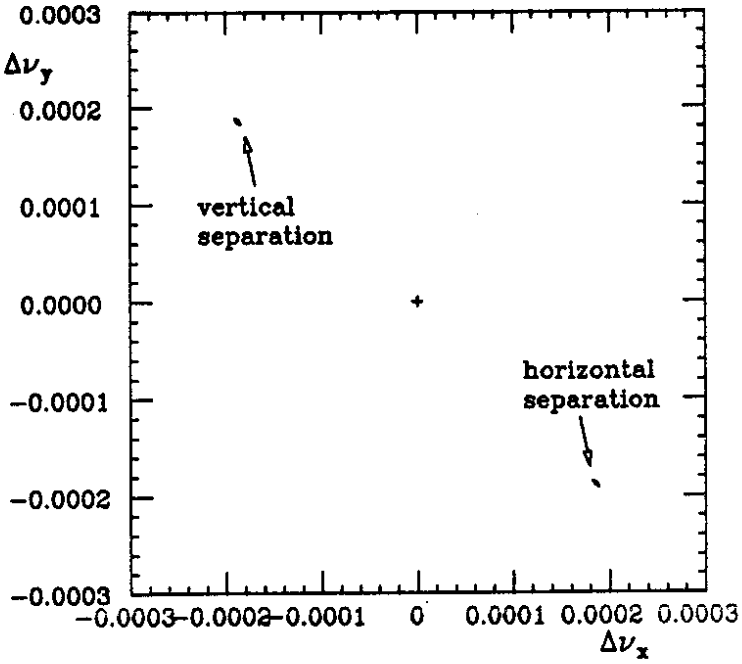

Figure 3.2 Long range tune shifts and spreads due to encounters in a high beta (10 metre) intersection region, with the same crossing angle $\alpha^* = 75$ microradians, so that the beam separation is $33.5\ \sigma$, horizontally or vertically.

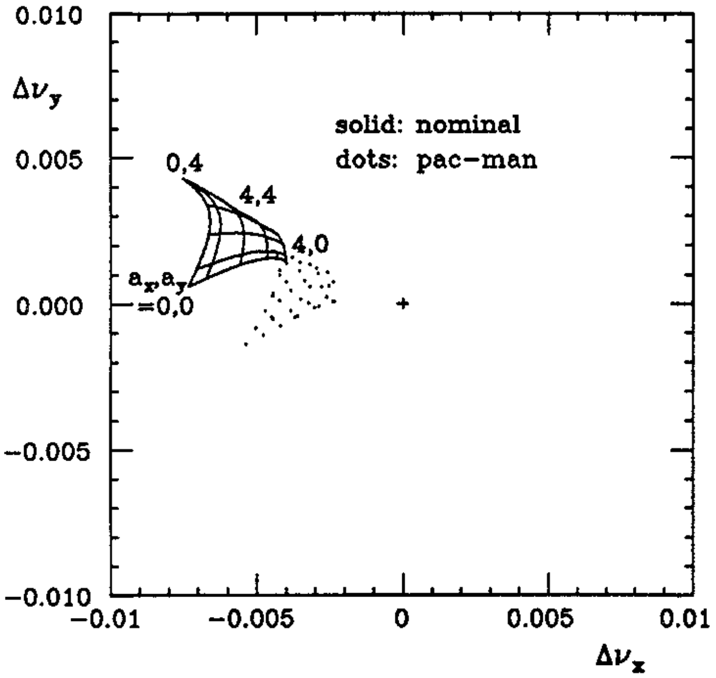

Figure 4.1 The total net tune shifts and spreads due to head on and longe range interactions in two low beta and two high beta collisions in a nominal SSC, with vertical separation in all intersection regions. PACMAN bunches experience half of the long range, and all of the head on, collisions.

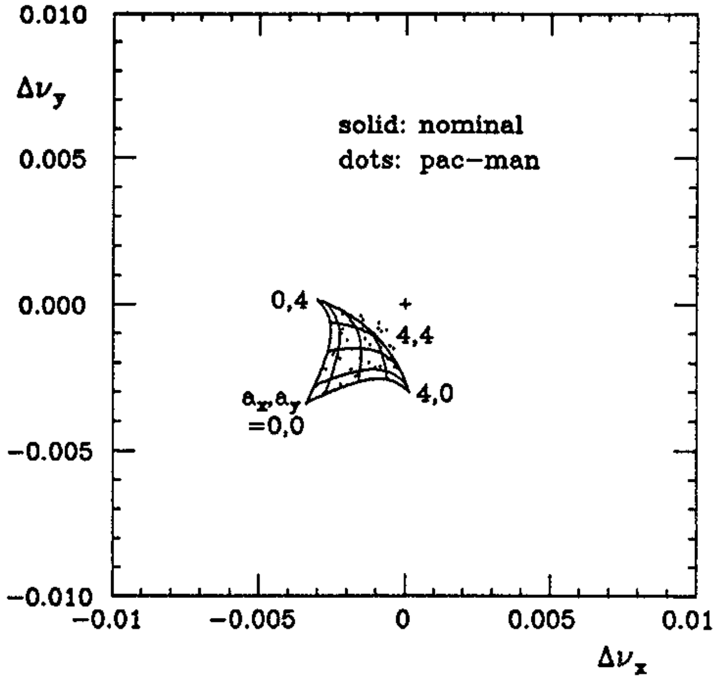

Figure 4.2 The total net tune shifts and spreads when alternate intersection regions have vertical and horizontal separations. PACMAN bunches now occupy the same tune region as the regular bunches, which have significantly reduced tune shifts, but the same tune spreads.